REVIEWS

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Fragments of Infinity, A Kaleidoscope of Math and Art. By Ivars Peterson. John Wiley & Sons, Inc., New York, 2001, vii + 232 pp., ISBN 0-471-16558-1, $29.95.

Reviewed by William Mueller

On a cold, rainy night in the early spring of 1952, Jackson Pollock stood in his candlelit barn, staring at a huge canvas spread out on the floor before him. His arms hung at his sides: a dripping paint stick in one hand, a glass of bourbon in the other. He was stuck. Although he had swirled the paint on the canvas into what was, by then, a familiar primordial pool—oily, organically colored, chemically redolent—he could not bring it to life. It was a problem, and no amount of half-light, drinking, or Coleman Hawkins, blaring from a spattered phonograph on the floor, offered any inspiration. As night turned to morning, Pollock sullenly confronted his capacity to see it through. He finally grew desperate, and summoned a friend and fellow painter. The peer review was merciless: "It looks like vomit" [**6**, p. 6].

Only a few years before, the solutions had come so effortlessly. Pollock had discovered a language of gesture and movement that had connected him immediately to his work, and he had produced, in quick succession, a forceful series of revelations that had astonished the New York art world. He'd managed to find, amid gallons of tangled house paint, a passage out of himself and across human boundaries. The paintings were received as confounding, confrontational, even shocking, but they earned Pollock a long-sought reputation, and they would change modern art. The affirmation, however, was ultimately unfulfilling. For Pollock, nothing could compare to the rapturous power of being inside a painting that was "working."

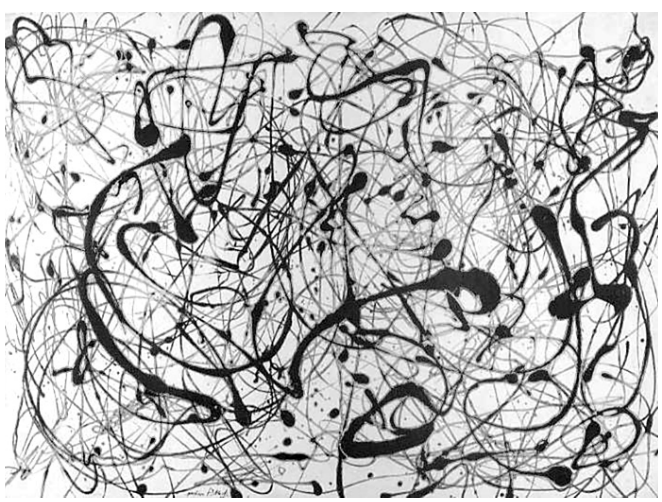

Figure 1. Number 14, 1948, by Jackson Pollock [**8**, p. 239].

Then, unaccountably, inspiration seemed to disappear. As Pollock struggled to regain the self-confidence of those transcendent seminal encounters, his vision became murky. The lost lucidity in his work slowly translated into an acute psychological distress. He scraped and reworked the canvas that had so vexed him on that rainy night, returning to it for over half a year. Eventually, he had had enough: he picked up a 2×4, slopped blue paint along an edge, and plunged it repeatedly into the wet canvas, varying the angles in some faint residual hope of artistic gesture. It had to have felt like a desperate act, to impose something as laughable as linearity on such a tormented landscape. Nevertheless, "Blue Poles" came to be recognized as an epic exposition of human limitations—a marvelous disaster. It would sell, after Pollock's death, for a record price of two million dollars at auction.

Mathematicians may relate to certain aspects of the story. It isn't difficult, after all, to draw parallels between the working artist and the working mathematician. Both endeavor to express connections among abstract manifestations of the most elusive of universal concepts. Both are dependent upon specialized formal languages, incomprehensible to the great mass of the uninitiated, the insufficiently devoted, or the simply unable. Both struggle with the finite forms and structures with which we are all destined to formulate our limited visions of the infinite and the ineffable. Both, finally, must defer to creative mechanisms beyond their complete control: intuition, aesthetic judgments, and the subjective regard of their peers.

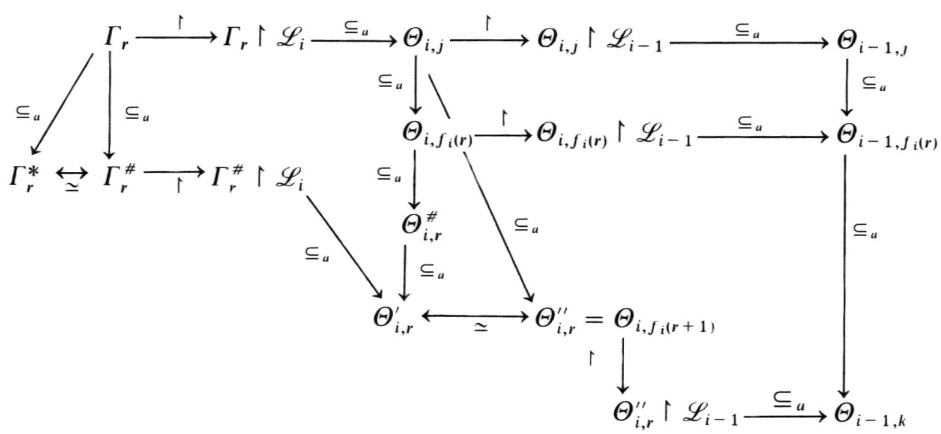

Figure 2. Figure 3.13, 1983, from *Degrees of Unsolvability*, by Manuel Lerman [**3**, p. 291].

It is, of course, also very easy to oversimplify these connections, leaving only a filamentary network of clichés. Not all artists are troubled alcoholics like Pollock, struggling to release something trapped within. Similarly, not all mathematicians are brilliant schizophrenics like John Nash, recognized for their idiosyncrasies as much as for their work. A persistent cultural fascination with the ultimately facile category of "genius," combined with a sentimentality that reflexively equates intellectuals with loveless loners, only perpetuates these popular myths. Recent Hollywood treatments of both Pollock and Nash [**1**], [**2**], for example, showcase troubled individuals engaged in solitary acts of revolutionary creation. The craft, dedication, society, and often quite impersonal inspirations that form the day-to-day life of most working artists and mathematicians are not the stuff of media stardom.

Ivars Peterson, a writer of popular columns on mathematics for both the MAA and *Science News*, has written a book that, by way of contrast with the Hollywood blockbusters, plainly presents a selection of contemporary work-a-day artists and mathematicians who look to each other for inspiration. Each of the ten chapters in *Fragments of Infinity* is organized around a shared mathematical theme, explored by a particular subculture at the math/art intersection. The themes are well-trodden in the popular mathematical literature—fractals, tilings, minimal surfaces, etc.—but Peterson distinctively finds artists who use these mathematical ideas as organizing elements and mathematicians who are inspired by their conceptions. In many cases, the math/art divide is convincingly rendered superfluous, both for the individuals portrayed and for the work that they produce. It is an engaging premise. Peterson is a winning writer, and he tells these stories of collaboration and cross-fertilization with an easy-to-read simplicity.

The book, however, contains very little in the way of actual mathematics. Mathematical notation is avoided completely, and there is no sign of computer code, despite many examples of artists working with computer-assisted designs. The author seems to assume that readers will come to the book with only the most rudimentary technical knowledge. Elementary terms such as "coordinate" and "plane" are patiently described, and the reader is taken deliberately through constructions like the "unusual" Möbius strip. Higher-level mathematical ideas, when they are approached, are handled poetically, in a style that is clearly directed at a "general" audience. There is nothing wrong with writing at this level, of course. One wonders, however, who this "general" audience might be, and why they would be reading this book. Mathematicians, certainly, will look through these pages and wonder if their subject is being faithfully represented.

Artists will undoubtedly have similar reservations. The book has been published with high production values, and it includes many fine photographs of artists' work. There is scant mention, however, of artistic materials and processes. There is nothing in the text that can really be called art criticism, either. In a typical chapter, standard artistic terms such as "conceptual" are explained, followed by anecdotes of artists who neatly fit the bill. Rather than providing satisfying analyses of artists' interactions with their work, or with other artists, the text offers only a collection of short, tidy images. The sculptor Helaman Ferguson, for example, is described this way: "amid the roar and the dust, artist and stone engage in intimate conversation—one that can readily break into song" (p. 12). The implied connection between the vibrations of the stone and the geometry of the emerging form has a mathematical resonance that seems far too convenient. Peterson often works in metaphors like this, at times saying more about his own search for expression than he does about the artist or their work. The liberties can become aggravating, as when Pollock's paintings are flatly misrepresented as "fractal splashes" (p. 2).

Peterson begins his book by saying that it is "about creativity and imagination at the intersection of mathematics and art" (p. v). He says that he will "highlight the processes of creativity, invention, and discovery intrinsic to mathematical research and to artistic endeavor" (p. vi). It is an ambitious declaration, and it sets expectations high. Indeed, throughout the book there is the recurrent suggestion that a grand connection between the two subjects will be revealed. The surface similarities between mathematicians and artists, already noted, cannot serve. What *Fragments of Infinity* seems to promise, tantalizingly if never quite explicitly, is some exposition of the *nature* of these similarities. The promise is never truly delivered.

Part of the difficulty is that the author has limited himself—unnecessarily—to representative examples of a predetermined theme. Sources come entirely from the visual arts, are disproportionately sculptural, and lean heavily on the immediate appeal of geometric forms. In an opening tour of the National Gallery in Washington, D.C., for example, the author stops at the entryway to admire the elegant curves of Henry

Moore's *Knife Edge Mirror Two Piece*, claiming for Moore a "fascination with holes and topological transformations of space" (p. 2). The author assiduously avoids walking around the building to confront Frank Stella's *Prinz Friedrich von Homburg, Ein Schauspiel, 3X*, the mathematical lessons of which would be far more difficult to encapsulate. (It is possible that the author's visit predated installation of Stella's work, but one might pick from any number of similarly perplexing pieces in and around the gallery.) Given such limited observations, it seems doubtful, right from the start, that the author will be able to generalize his conclusions convincingly to all of art and mathematics, however suggestively they may be presented.

Peterson has put himself face-to-face with the fundamental impediment to popular writing: To write an account of a difficult subject, it becomes necessary to accommodate the lay reader by leaving out most of the difficult things. When these difficulties are an essential, even a defining, characteristic of the subject, a thorny representational dilemma presents itself. The problem, of course, is only compounded when two difficult subjects are treated simultaneously. Art and mathematics both challenge us to express difficult things—concretely, exactly, and in a language that is appropriate to the task. Peterson has made a commendable effort to hurdle over this impediment with stories, metaphors, and poetics, but the missing complexities—the mortar that holds these subjects together—finally make *Fragments of Infinity* impossible to assemble.

There is another difficulty inherent in Peterson's approach, a deeper one. It arises from the casual application of an unexamined system of equations relating the structures and forms of mathematics with those of the artist. In simplest form, the presence of structure in an artwork, *in itself*, is equated with mathematical intent. Peterson ably demonstrates that mathematical structures can lead artists to a certain kind of compositional discipline. Strict mathematical discipline, however, is antithetical to many of the most fundamental tenets that have informed modern art. The Soviet painter and designer El Lissitzky was very explicit about this: "The parallels between Art and mathematics must be drawn very carefully, for every time they overlap, it is fatal for Art" [**4**, p. 348]. Later, in another context (but just as typically), the American painter Robert Motherwell wrote that "... modern painting ... is symbolic and poetic, not discursive and descriptive ... the latter is always trying to infiltrate modern painting, usually under the tag of some 'humanism' or another. What shit!" [**5**, p. 155].

There is a long history of popular accounts that link contemporary art to contemporary ideas in math and science. From parlor primers to magazine articles to coffee table books, they have usually been written in the service of some sort of democratic "popularization," and the associations that they make have almost always been exaggerated. Lissitzky, for example, has been persistently aligned with explorations in multidimensional geometry made by his mathematical contemporaries. Lissitzky's socialist perspective, and his deep involvement with the history of graphic design—both probably more influential in his art—only confuse easy-to-digest stories of his mathematical vision. An example in recent years is the spate of books, posters, and T-shirts extolling the wonders of "fractal art," which marketably replace the subtleties of art history with the inexpressive complexities of form.

The truth is that when artists order their work, their notions of harmony, unity, and precision often come from sources that are anything but mathematical. Take, for example, an artist that Peterson features in *Fragments of Infinity*. Arlene Stamp was a high school mathematics teacher before turning to art full-time. She won a commission to create a mosaic for Toronto's Downsview subway station by suggesting a nonrepeating pattern "based" on the digits of π. The conception of the design proceeded along the following lines. First, an initial state of the mosaic was imagined, consisting of a sequence of rectangular units, side by side, each ten tiles wide. In the next stage, Stamp imagined each rectangular unit shifted over the previous unit by a number of tiles rigorously determined by the sequence of digits in π's decimal expansion. Finally, the amount of overlap occurring along any column in the design was to be translated into a particular color of tile. This would all seem very algorithmic, if Stamp had followed the instruction set—but she didn't. During the construction, she found it necessary to incorporate a variety of inexact human gestures to express her ideas. Peterson tells us that "some regions of overlap may end up as many as three or four layers thick" (p. 92). Stamp, however, "used four sets of eight colors for the project, deploying them in different ways on surfaces in various parts of the station" (p. 92). When the final mosaic is considered, we must give Stamp her due credit as an artist. Only then can we appreciate the many choices that she has made—to arrange the tiles, to shift the tiles, to count the overlaps, to choose the colors in various places—that have no basis in mathematics. The mosaic, in the end, is not strictly about π. Stamp herself says: "I am interested in the beauty of embedded possibility"(p. 94).

Many other artists could serve as examples of the kinds of mistaken impressions that can arise from a too-easy calculus of form and mathematics. Ellsworth Kelly, a contemporary painter, would seem to fit nicely among the artists in Peterson's book. Kelly's paintings are simple and geometric, composed of curving fields of primary color, expressing their exuberance through the exquisite tensions of tangency between various curves in the paintings and between the curves and the edges of the canvas. The carefully poised kisses between Kelly's forms gives the viewer the same kind of "Oh!" that a mathematician might experience when discovering a uniqueness theorem. Kelly's great gift is his ability to see these forms, in all of their everyday manifestations. He has painted plants, shadows, reflections, bridge arches, and hills, all in the same hard-edged geometric distillations. The colors with which he floods his geometry express the sheer joy of seeing. If Kelly is "mathematical," he has invented the term for himself. His abstractions from the natural world are something that we respond to intuitively, in a way that a purely mathematical art could never achieve.

Jasper Johns, another contemporary painter, offers further illustration. Johns has painted the numbers from one to nine, and has composed many other paintings around various organizations of these digits. Is Johns interested in number theory? It's doubtful. What the numbers represent, in Johns's language, is form itself, made visible through familiar, everyday exemplars. The forms that Johns chooses are often iconic, suggesting larger constraints in our everyday life. He is famous, for example, for having painted the American flag. Johns's paintings struggle with these forms, working with thick layers of paint, in a discourse that is inward and painterly as much as it is outward and political. To focus on the underlining lines, rectangles, and forms of Johns's *Flag*—to call them "mathematical"—would reduce the painting, quite inaccurately, to the clean lines and assured colors of a "United We Stand" bumper sticker.

Perhaps, however, the difficulty of drawing too close a correspondence between art and mathematics is best illustrated by considering the set difference—the artists and mathematicians who would not sit comfortably in any grand commonality, however much they may appreciate the work that is accomplished in the opposing sphere. Among mathematicians, this might include a large group of applied practitioners, many formalists (though I certainly know logicians who revel in the visual beauty of their symbolism), and certain Platonists, inspired by forms that they perceive to be super-human, beyond the lesser, personal impulses of art.

Among artists, the avant-garde pioneer Robert Wilson comes to mind. Known for his epic theatrical productions, especially for the stage designs of his many operas, Wilson has also found expression through drawing, painting, and sculpture. A recent installation piece, *14 Stations*, combines all of these influences. Wilson says of it [**9**, p. 3]:

> My work is an environment, an installation that brings together elements of architecture, sculpture, art, music, and language. In a certain sense, it is a mental landscape. Call it an encounter of different cultural traditions, in which I have tried to invent my own language .... I always work with a horizontal line, which stands for time, and a vertical line, which, for me, always means space. This is something personal ....

The vertical line, in this case, is a boardwalk that takes viewer past a series of fourteen small cottages, arrayed to either side. The interior of each cottage is examined through a small window, admitting only a single viewer at a time. What one sees, hears, and smells, in each case, is a carefully choreographed tableau—completely outside of one's waking experience—that immediately strikes at the gut. The cumulative effect, as one wanders the entire installation, is undeniably affecting, though it is difficult to make much "sense" of the reaction. Wilson's relation to the formal order of the piece, much of it appropriated from religious tradition, is ambiguous and unsettling. Taking Johns one step further, Wilson uses the formalism to question, and perhaps even repudiate, its very idea. Robert Wilson is not a mathematician.

Artists are constantly struggling with the limits imposed by their media. Painters like Pollock, Kelly, and Johns alternately fight, then celebrate, the restrictions of their flat, framed arena. Musicians—like Coleman Hawkins—struggle within traditions of composition and performance that, at the same time, carry with them the rich formal history of their music. Writers, from Pushkin to Pynchon, reinvent literature, but do so on a stage that has been set by generations. Mathematicians, also, must work within their subject's history and limitations. Every mathematician has experienced a surprising counterexample, seen a conjecture disproved, and had to reconsider the outlines of the dimly lit subchamber of mathematics they call their own. Turing and Gödel, and the complexity theorists who have followed, have made fundamental limitative theorems a fact of mathematical life. Still, like artists, mathematicians regroup, consider their progress, and move on, exploring new directions and new possibilities.

Artists remind us of another limit—our own. Whatever medium is chosen, and whatever forms are imposed, expressions of our collective human understanding never feel finally, satisfyingly, complete. Pynchon writes metaphorically of Mason and Dixon and their army of axe-wielding Colonials plowing through the mysteries of the American wilderness at precisely 39° 43′ N [**7**]. Like Pollock's blue poles, the meager structures that we force upon a world that is so vastly different from ourselves may seem laughable. It would seem that way, at least, if those structures weren't such a fundamental part of who we are.

Peterson quotes the geometer H. S. M. Coxeter, saying of visualizations in chaos theory that "It makes it almost embarrassing for abstract artists to do abstract art because these are things that are so varied and so beautiful that one doesn't need to go any further" (p. 133). This is chutzpah, on a cosmological scale. There is always further—infinitely further. Artists and mathematicians know this in their bones. Together they survey the universe, extending away from them in myriad dimensions, and from within the tiny perimeter that they are able to carve out, they express what they can see, and what others may see. It is this shared sense of place and purpose, perhaps, that connects artists and mathematicians most fundamentally.

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Mathematical Models in Population Biology and Epidemiology. By Fred Brauer and Carlos Castillo-Chávez. Springer, New York, 2001, xxiii + 416 pp., ISBN 0-387-98902-1, $59.95.

Reviewed by **Shandelle M. Henson**

Recently I stood with mathematician Jim Cushing and ecologist Bob Costantino in the cool dimness of the Mt. Wilson Observatory, looking across the railing into the telescope room. After a period of silence, Bob turned to us and said, "Astronomers looked up at the sky and assumed there was order. So they formulated and tested mathematical models. Ecologists look out at nature and say, 'This stuff is too complex to explain and predict.' That's self-defeating. Surely there is enough low-dimensional order out there to allow prediction of ecological dynamics. When the mindset changes—as it surely will—scientific progress will follow."

As I consider recent advances in ecology, I have the growing sense that these words may be prophetic. Ecology may well stand at the threshold of an enormously productive mathematical revolution.

For those unfamiliar with the issues, some explanation is in order. Mathematics and ecology have had an uneasy relationship. It is true that each discipline has benefited from the other. On the one hand, the models and questions of ecology have contributed substantial motivation to the mathematical theory of dynamical systems. On the other hand, mathematics has contributed a number of important theoretical insights and tenets to ecology. However, actual quantitative connections between dynamic models and data have been scarce. While the discipline of physics has long embraced mathematical models and controlled laboratory experiments as primary tools for the explanation and prediction of dynamic physical phenomena, ecology has been slow to follow.

In fact, the hypothesis that population fluctuations are shaped largely by low-dimensional deterministic forces has caused considerable controversy for nearly a century. During the last few decades, however, this hypothesis has been rigorously and successfully tested in laboratory populations through the application of dynamical systems theory and statistics. Careful studies involving mathematical models, controlled laboratory population experiments, and statistical techniques have unequivocally identified many low-dimensional deterministic phenomena in population data. These phenomena include equilibria, cycles, transitions between dynamic regimes (bifurcations), multiple attractors, resonance, basins of attraction, saddle influences, stable and unstable manifolds, transient phenomena, and even chaos. Robust qualitative and quantitative predictions have become possible for several laboratory systems; see [**1**]–[**5**], [**8**], and [**10**]–[**12**].

A major goal of laboratory studies, of course, is to gain clear insights that might be applied to fluctuations in field populations. Despite the very real difficulties of developing quantitatively accurate models for field systems, many researchers are optimistic that we are gaining the necessary conceptual tools and insights. If some of the recent successes in the laboratory can be extended to the field, unprecedented advances in field ecology may lie just around the corner.

So what does this have to do with textbooks for mathematical modeling? In this exciting climate of accelerating change, students of biology in general and ecology in particular should be trained in the mathematical methods just as physics majors are. Interdisciplinary courses on mathematical models in biology are springing up at many university campuses. These classes are important to the future of the discipline of ecology. Not all the students thus trained will go on to do mathematical modeling in their careers; but hopefully they will have lost any prejudice they might have harbored against the method of abstraction and will point their own students to the importance of mathematical training. In other words, classes in mathematical modeling can help change the academic culture of biology and ecology departments.

I have had the pleasure of teaching such courses at the College of William and Mary and Andrews University. The subject seems to be popular, and it has attracted some excellent students. We cover the basics of deterministic discrete- and continuous-time linear and nonlinear models, both scalar equations and systems. Topics include analytic solutions of linear equations, equilibria, linearization, stability, phase portraits, bifurcations, simulations, and modeling methodology. We spend a good deal of time discussing the philosophy of science: how are mathematics and science different, how are they similar, and how should mathematics be used in science? We talk about logic, epistemology, and various notions of certainty. The students become familiar with the literature, work together in interdisciplinary research groups, and learn to give research talks. It would be nice to run a second semester of the course, covering issues of stochasticity, parametrization, validation, and the connection of models with data. Teaching this course has been fun and rewarding.

It has also been a challenge. Frankly, teaching a good interdisciplinary course in mathematics and biology can be tough. It seems to me that the ideal classroom is a mix of biology and mathematics majors. Each discipline learns to respect the other; the mathematics students learn some biology and the biology students some mathematics; and all of them get a taste of the exciting synergy of interdisciplinary collaboration. At any rate, even if one did wish to separate the biology and mathematics students, many universities do not have the resources to run two such courses. The problem with having a mixed clientele, of course, is pitching the material at the right level of mathematical difficulty. Usually the biology majors will have had one or two semesters of calculus. Mathematics majors who are drawn to such a course, in contrast, tend to be more advanced in their mathematical curriculum; they are often seniors looking for an interesting mathematics elective.

Although the inequalities of mathematical background present a real opportunity for an intellectually invigorating classroom, they can also create various sorts of problems. Feelings of insecurity and attitudes of disdain among the students are not uncommon. The biology students who take such a course are usually pretty serious and sometimes even intellectually passionate; but they often feel insecure about the mathematics. And, it is pretty common for one or two lazy or anti-intellectual mathematics majors to enroll just because it sounds like an easy elective. These students sometimes attempt to cloak a refusal to learn with a mantle of mathematicians' disdain. (This is easy to see through, but it certainly is annoying.) It is often unclear how fast the instructor should go through the material, and to what depth; and each group of students may have different needs in this regard.

How should a mathematics professor deal with such problems? I have tried various methods. Most importantly, one must create an atmosphere of interprofessional respect. It helps if the professor has credibility as an interdisciplinary researcher and collaborates with colleagues from biology. It is also helpful if biologist colleagues sit in on the course, or team teach it, or give guest lectures. I try to cover the biology as well as the mathematics and to present the modeling techniques in a unified scientific framework. Along with respect, it is important to create an atmosphere of security. Biology students must be certain that it is okay to ask questions about the mathematics, that there is no reason to be ashamed because they don't know as much math as the mathematics majors. They need to be encouraged to jump in with both feet, simply learn as much as they can, and be intellectually passionate as scientists. The mathematics majors should be encouraged to learn some science, and to dig deeply into some of the fascinating and difficult mathematical topics (such as chaos) that come up in biology.

Traditional lecturing—normally my most effective teaching mode—must proceed at a more leisurely pace, and can be punctuated by frequent "breaks." Students come up to the board to work problems, we discuss issues of human population growth, I probe the students' understanding of various issues by calling on them individually. Sometimes when going through a long algebraic derivation, I will ask each student in turn: "Clara, what is the next step in solving for λ?" "Matt, what does the eigenvalue tell you about the dynamical system?" When one has a good feel for the level of each student, one can usually ask questions to which he or she can give appropriate and substantive answers. These frequent changes of pace keep the attention of those with more mathematical background and help those with less to absorb the material. Because of the dampening effect of lazy students who are not really interested in the subject, I have started personally interviewing students before they begin the class. I tell each student what the class will be like and what I expect in terms of participation and intellectual engagement. I also warn that the nontraditional nature of the class might make it a miserable experience for someone who is not excited about biology.

Finally, there is the problem of choosing an appropriate textbook. At first I imagined I would simply choose a textbook and follow it, as we tend to do in mathematics. Edelstein-Keshet [**6**], perhaps the main workhorse in the area, nicely integrates the mathematics with the applications and brings together a treasure-trove of material. It has a prerequisite of "basic calculus," but it was too difficult for my students who had done well in a year of calculus at William and Mary. Hastings [**9**] requires a year of calculus and suggests having some previous exposure to ecological ideas. This well-written book was created for Hastings's population ecology students, and it is a favorite with my biology majors. It is too elementary in terms of computation and theory for mathematics majors, although it provides excellent supplemental reading for these students.

The book under review is a new entry in this field. Fred Brauer and Carlos Castillo-Chávez have written a solid, comprehensive book organized around the three topics of single species models, interacting species models, and structured population models. They cover discrete and continuous time equations, linear models and linearization, qualitative analysis and phase space, bifurcations, and delay equations. Biological applications and classic topics include epidemiology, vaccination schemes, harvesting, delayed recruitment, Lotka-Voterra models, chemostats, competition, predator-prey systems, mutualism, Kolmogorov models, invasion and coexistence, the community matrix, and age structured McKendrick-Von Foerster models (including numerical schemes). Some chapters include case studies of such topics as the eutrophication of a lake, oscillations in flour beetle populations, Nicholson's blowflies, and the spruce budworm. Most chapters contain several interesting projects; for example, estimating the population of the U.S.A. and models for blood cell populations, neurons, and pulse vaccination.

The book reads as a well-written and fairly traditional undergraduate mathematics textbook, with theorems and some proofs (although many theorems are stated without proof). Its prerequisites are "a year of calculus, some background in elementary differential equations, and a little matrix theory." It would work well as a text for an upper division undergraduate topics course in applied dynamics, or as a graduate course for mathematically advanced ecology students. It served as an excellent reference and source for problems and projects in my own undergraduate interdisciplinary class. However, my students found it more difficult than Edelstein-Keshet [**6**].

The conclusion of the textbook hunt for my particular situation has been the following: (1) the kind of course I want to teach is too fluid to run in lockstep with a textbook; (2) no book will be at the right level for all the students in my class; indeed, there is no "right level"; (3) textbooks are useful for assigning readings and problems, as sources for student projects, and as reference books for the scholarly libraries of my upcoming young research biologists and applied mathematicians. In the Spring 2002 semester I used two texts: the book under review and Hastings [**9**]. I assigned readings and homework problems out of both books as appropriate, but did not base my lectures on either book. Instead, I ended up writing my own set of notes tailored to the interdisciplinary mix of students. This approach seemed to work well.

Brauer and Castillo-Chávez write in the preface: "This book is intended to inspire students in the biological sciences to incorporate mathematics in their approach to science .... A secondary goal is to expose students of mathematics to the process of modeling in the natural and social sciences." This statement cheers me, and I am reminded of the words of the evolutionary statistician R. A. Fisher [**7**, p. ix], when he said of mathematics and biology: "I can imagine no more beneficial change in scientific education than that which would allow each to appreciate something of the imaginative grandeur of the realms of thought explored by the other."

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