Mathematical *Wunderkammern*

William Mueller

1. INTRODUCTION: THE AGE OF WONDER. A giddy craze was sweeping across Europe at the turn of the 17th century. The wealthy and the well-connected were hoarding things—*strange* things—into obsessive personal collections. Starfish, forked carrots, monkey teeth, alligator skins, phosphorescent minerals, Indian canoes, and unicorn tails were acquired eagerly and indiscriminately. Associations among these objects, if they were made at all, often reflected a collector's personal vision of an underlying natural "order". Critical taxonomy was rarely in evidence.

The enthusiasm that characterized such collections was captured by Francis Bacon [**1**, p. 247], who ironically advised "learned gentlemen" of the era to assemble within "a small compass a model of the universal made private", building

> . . . a goodly, huge cabinet, wherein whatsoever the hand of man by exquisite art or engine has made rare in stuff, form or motion; whatsoever singularity, chance, and the shuffle of things hath produced; whatsoever Nature has wrought in things that want life and may be kept; shall be sorted and included.

These "cabinets" were proudly displayed, and selected visitors were allowed to wander through the accumulated "shuffle of things" to express their awe and admiration. Other expressions, such as perplexity and incomprehension, were circumscribed by what was, in pre-Enlightenment Europe, a well-practiced deference to presump-

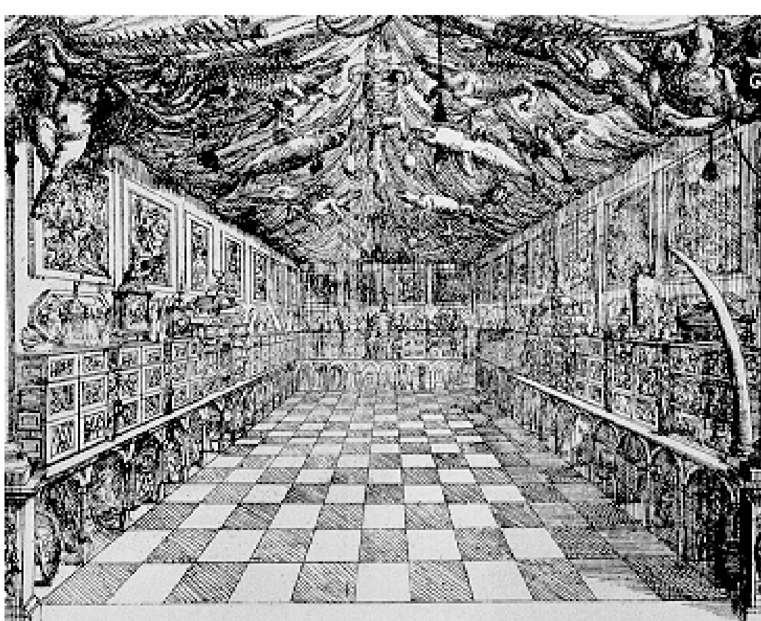

Figure 1. The museum of Manfredo Settala, Milan, ca. 1600.

tive authority. Indeed, the cabinets can be seen as a sort of physical manifestation of centuries-old religious and philosophical states of consciousness. These first "museums" presented visitors with an opportunity to place the era's flood of information within a comfortable psychological order. As the century progressed, patrons would pay for the privilege of a carefully arranged understanding of the world's troubling uncertainties.

Europe in 1600, one must remember, was crossed by many unsettled intellectual currents. The Renaissance, which had begun in Italy in the 14th century, had spread throughout Europe by the end of the 16th, reviving the continent's slumbering passion for art and learning. As the Renaissance progressed, however, it had transformed Europe into a much more complicated place. By 1600, the consequences of so much concentrated inquiry were beginning to show in the troubled creases across many European brows. Explorers were regularly bringing back incredible, inscrutable objects from the New World and the Far East; objects whose existence brought into question the centrality of Europe and the primacy of its culture. Simultaneously, the Scientific Revolution, led by the likes of Copernicus (1473–1543), Bacon (1561–1626), and Galileo (1564–1642), was challenging the centrality of the world itself. As the extraordinary scope of Geography and Nature began to emerge, many traditional, comforting notions seemed to shrink in comparison. It was a bit too much for some, and the profound unease stirred up by such revolutionary changes gradually fueled a reaction. In extreme cases, science was labeled Heresy. (Giordano Bruno was burned at the stake in 1600; Galileo was imprisoned by the Inquisition and threatened with torture in 1633.) Scientists, still in their early days of mixing powders and picking apart anatomies, were not yet in a position to respond with assurance.

What is remarkable about this era is that, even when every boundary seemed to be expanding beyond the known horizons, and even when the objects from those far horizons suggested a world that was larger than the imagination, there was nevertheless a desire to reach further. It was daring, for what lay beyond was completely unknown. Commerce, politics, and religion, each with its own agenda, tried to temper the excessive enthusiasm. Nothing, however, could direct the collective fascination with the New World's dark tumult. Curiosity cabinets proliferated across Europe. Eventually, these cabinets became the first museums of science, natural history, medicine, and art, depending on how curators chose to arrange their accumulated treasures. What these first museums showed their patrons was that the New World—whatever it was—was undoubtedly *wonderful*. The collections became known as *wunderkammern*, or "wonder cabinets".

The emerging scientific community wasn't entirely pleased with the craze for *wunderkammern*. Those who took the Revolution seriously found the curiosity cabinets not only frivolous, but reactionary—an impediment to the properly measured progress of scientific reason. Wonder, they argued, ought to be lavished on the technical advances in navigation that had allowed explorers to sail back and forth to the New World time and again, not merely on the gewgaws that they had brought back. Galileo [**8**, p. 459] wrote with apparent distaste of the "curious little men" who would gather

> ... a petrified crab, a desiccated chameleon, a fly or spider in gelatin or amber, those small clay figurines, supposedly found in ancient Egyptian burial chambers. ... Our poet errs as much as would a painter who, purposing to depict a particular hunting scene, were to clutter his canvas with conies, hares, foxes, goats, deer, wolves, bear, lions, tigers, boars, hounds, greyhounds, leopards, and all manner of wild beasts, clustering at will animals of the hunt with every sort of game, such as to liken his painting more unto a representation of the entry into the Ark of Noah than unto a natural hunting scene.

Descartes [**9**, p. 20] gave the curiosity cabinets an even more peremptory wave: "What we commonly call being astonished is an excess of wonder that can never be otherwise than bad."

2. MATHEMATICS IN THE AGE OF WONDER. Mathematics, as might be expected, was an eager participant in the excitement that marked the beginning of the 17th century. By this time, a tremendously fruitful interchange had developed among mathematics, science, engineering, and commerce. Practical problems that arose in the burgeoning complexities of international trade, in navigation, in the construction of cities, and in the logistics of warfare were leading to fundamental new developments in algebra, geometry, and computation. As the century progressed, mathematicians such as Harriot and Oughtred would rediscover the algebraic techniques that Arab mathematicians had employed during Europe's Dark Ages, rewriting them in a new language of symbols (much to the chagrin of typesetters); Desargues, Pascal, and Descartes would completely re-invent the way in which geometric problems were investigated; Napier would introduce an efficient new tool for computation called the logarithm; and Fermat would apply infinitesimal methods to the problem of finding the maximum and minimum values of a function. By the century's end, all of these developments would coalesce into one of mathematics' most lasting contributions to practical science: The Calculus.

These contributions were widely noticed and appreciated by the 17th century public. Indeed, "mathematical" apparatus such as calculating machines, drawing instruments, astrolabes, sundials, and the recently invented sector and slide rule found their way into many of the cabinets of curiosity. Unnoticed by much of the public, however, was mathematics' slow turn inward. This was, after all, the "early modern" period of bold new experiments in abstraction. Mathematical advances gradually became much less generally understood, even by those who considered themselves to be active participants in the era's intellectual life. The logic and systematic discipline within mathematics, as well as the many practical applications that it produced, continued to impress, but outsiders could no longer hope to share in the excitement of fundamental new treatises the way they might have only half a century before. As a result, there was not a rush to learn logarithms the way that there was a rush to taste new teas from China.

Many mathematicians and scientists of the Revolution found this to be an entirely appropriate state of affairs. Mathematics and Science, as they were being newly imagined, were disciplined endeavors proceeding along ineluctable paths of reason. Mathematics, in particular, was not advanced through the wondrous contemplation of numerical or geometrical gewgaws. Descartes [**2**, p. 47] saw mathematics as an exercise in intellectual discipline, necessary for discovering unimpassioned truth. He wrote:

> As for myself, who am conscious of my feebleness, I have resolved to observe constantly, in the search after knowledge, such an order that, commencing always with the most simple and easy things, I never take a step forward in order to pass the others, until I believe that nothing more remains to be desired concerning the first.

Descartes was removing himself as far as he could from the unrestrained enthusiasms that characterize wonder, and his sentiments were typical among mathematicians of the time. This view of mathematics, as a strictly logical construct with little tolerance for wanderers, would persist for centuries to come.

Descartes' resolve "to observe constantly ... an order" must be contrasted with the obsessive "observations" of the *wunderkammern* enthusiasts. Pascal [**2**, p. 58], who like Descartes cultivated more pious enthusiasms, remarks that:

> ...it is rare that mathematicians are observant, and that observing minds are mathematicians. ...Mathematicians who are mere mathematicians, have thus their understanding correct, provided always that everything be well explained to them by definition and principle: otherwise they are false and insupportable; for they are correct only on principles. And minds of observation, if only observant, are incapable of the patience to descend to the first principles of matters of speculation and imagination, of which they have no experience...

In other words, mathematicians, when doing mathematics properly, keep their heads down and their vision focused: A conception of mathematics as a terribly sober business.

It was, however, a conception that was undeniably productive. The historian Morris Kline [**14**, p. 391] has written:

> By the end of the [17th] century, mathematics had undergone such extensive and radical changes that no one could fail to recognize the arrival of a new era. The European mathematicians produced far more between about 1550 and 1700 than the Greeks had done in roughly ten centuries.

Many mathematicians were eager to see these accomplishments included among the "wonders" of the day. Pascal, for example, did not object to having his adding machine appear among the crocodile skins of many *wunderkammern*.

Most of the objects on the mathematical horizon in 1600, however, had been in sight for some time: algebraic equations, rows of calculations, trigonometric constructions, smoothly varying plane curves. Mapping the mist-covered routes between these objects, and negotiating the rocky shores that appeared when the mists began to clear, surely made for many satisfying mathematical voyages. The existence of the routes themselves, however, could not have been in serious doubt, and the voyages must not have seemed especially risky. Much of the time in mathematics, it was if western routes to the Indies were being found as expected.

It is thus difficult to describe the mathematics of the early 17th century in quite the same terms as those used to describe the revolutionary intellectual discoveries shaping the larger culture. The new era in mathematics that would come to pass by the end of the century was the result of a long, deliberate surveying of the mathematical landscape. There was no Magellan, Drake, or Hudson to bump into calculus; no Bacon, Hobbes, or Locke to repudiate the entire inherited intellectual system; no Copernicus, Galileo, or Kepler to re-imagine the mathematical universe. The mathematical discoveries of the early 17th century, though profound, did not explode ontological categories.

In the Age of Wonder, mathematicians and non-mathematicians alike looked past their astrolabes and into the heavens; beyond the era's navigational achievements and into the dark forests of the New World. Whatever Pascal may have wished, his adding machine could not compare with the awe, astonishment, surprise, and fear—the *wonder*—that these places evoked. Mathematicians focused on the well-established pleasures of their craft, and it would be some time until their cabinet was filled with curiosities that could capture the imagination in the same way as horned fish, fluorescent birds, and the artifacts of entirely unsuspected parallel cultures.

3. WONDERFUL MATHEMATICS. Henle [**11**] has pointed out that mathematics seems to pass through its own great Ages, corresponding in spirit to great Ages in art and culture, but often with a considerable delay. Descartes' influential belief that mathematics proceeds by its own internal logic, on its own time, independent of *a posteriori* input, may help to explain this apparent lag behind the *Zeitgeist*. Mathematics did eventually experience a comparable Age of Wonder, but it was some 200 years after Europe had been so transfixed by *wunderkammern*.

It was the 19th century when mathematicians *really* began to wonder. The beginning of that century was marked by the appearance on the mathematical horizon of the first ship from what was undeniably a New World: Fourier's 1807 publication of *Theory of the Propagation of Heat in Solid Bodies* [**6**, vol. 2]. The objects within its hold, which Fourier called "functions" but which did not seem to be anything of the sort, provoked an intense period of re-examination among mathematicians. Fourier's functions seemed to be perfectly well-defined by a method that was by then common practice—infinite series—and yet they exhibited properties that could not be reconciled with the predominant mathematical worldview. The series he used were trigonometric series, rather than the familiar Taylor series. The functions they defined did not seem to be determined by their derivatives, as Taylor series were. They were defined on closed intervals, yet they behaved as if they were periodic. Their graphs did not even *look* like functions—they jumped from one place to another without passing through intermediate values. All of these properties were very abnormal, and probably, it was thought, the result of some fundamental misconception. The new objects were dismissed by Descartes' disciples, who felt certain that this infidel mathematician and his ungodly "discoveries" could be explained away. Excruciatingly, however, they could not. Despite the best efforts of the Mathematical Inquisition, the terms of damnation could not be agreed upon. Instead, mathematics worked itself into a very non-Cartesian state of distraction. As the threads of Fourier's arguments were pulled apart, they tangled around the feet of everything that was holy. Edicts and proclamations were drawn up, but they could not dispel a profound anxiety that was spreading throughout the whole of mathematical culture.

It is a credit to the great mathematicians of the era that they, like their forebears from centuries before, responded to the challenge with a sense of wonder. Lagrange, Abel, and Cauchy began to dissect the new functions with a spirit reminiscent of the early anatomy theaters. Before long, other explorers, map-makers, and curiosity collectors would follow them on long voyages to a fertile new place called Analysis. Younger mathematicians would come to their museums to ponder the skeletons of strange new structures. They would wander through the curiosity cabinets of Cauchy's *Cours d'analyse* [**3**, series 2, vol. 3]. When it was their time, this next generation would follow curiosity's trade winds to an entirely New Mathematical World.

Undoubtedly, the second half of the 19th century was a time of tremendous mathematical treasures—*wonderful* treasures. Cantor, returning from his solitary voyages, brought back the varieties of infinity; Riemann sailed geometry to the higher dimensions where it blends seamlessly with analysis; Weierstrass charted the ever-stranger continuous but nowhere-differentiable functions, and Frege explored the deep caverns of logic itself. What mathematics *was*, at this point, became a thing of wonder in itself. The subject no longer seemed to proceed along Descartes' ineluctable paths of reason, but rather through flights of the imagination, inspired by dreams of what *might be*. Hans Hahn [**10**, p. 1956] later called this time a "crisis in intuition", and indeed it was; but it was a crisis only for those who could not leave behind the solid ground of Cartesian certainty. For those who were willing to test the wild waters of the great new sea and give up their minds to the uncharted, it must have been a wonderful, heady ride.

Consider, for example, the following testimonial from Sylvester, delivered in an 1869 address [**17**, vol. 2, p. 654]. This was before many of the greatest explorations of the era, but already it reflects a significant sea change away from Descartes and a strictly *a priori* view of mathematics. The tone suggests the contentiousness of those uncertain times.

> We are told that "mathematics is that study which knows nothing of observation..." I think no statement could have been more opposite to the undoubted facts of the case; that mathematical analysis is constantly invoking the aid of new principles, new ideas and new methods, not capable of being defined by any form of words, but springing direct from the inherent powers and activity of the human mind, and from continually renewed introspection of that inner world of thought of which the phenomena are as varied and require as close attention to discern as those of the outer physical world, ... that it is unceasingly calling forth the faculties of observation and comparison, that one of its principal weapons is induction, that it has frequent recourse to experimental trial and verification, and that it affords a boundless scope for the exercise of the highest efforts of imagination and invention. ... Were it not unbecoming to dilate on one's personal experience, I could tell a story of almost romantic interest about my own latest researches in a field where Geometry, Algebra, and the Theory of Numbers melt in a surprising manner into one another.

As analysis began to mix inextricably with geometry and the other branches of mathematics, the curiosities multiplied. New results stretched the limits of imagination. Consequently, mathematicians began to build intricate models out of wood, string, and plaster, transforming the far horizons of their explorations into finite forms that others might contemplate.

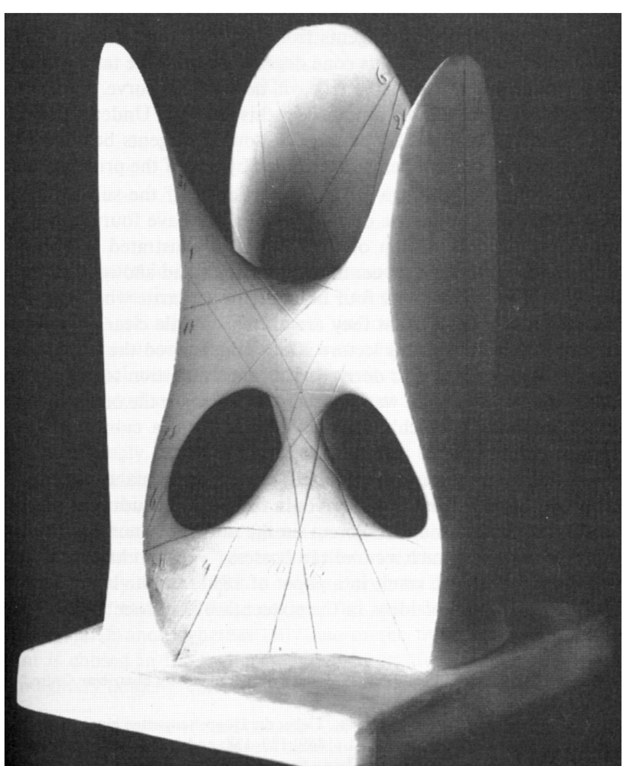

Figure 2. Clebsch's Diagonal Surface: *Wonderful*.

Fourier himself would have appreciated the spirit in which these models were created, for he had always believed in the power of mathematics to elucidate what could not be experienced directly. He had written [**7**, p. 8]:

> ... if man wishes to know the aspect of the heavens at successive epochs separated by a great number of centuries, if the actions of gravity and of heat are exerted in the interior of the earth at depths which will always be inaccessible, mathematical analysis can yet lay hold of the laws of these phenomena. It makes them present and measurable, and seems to be a faculty of the human mind destined to supplement the shortness of life and the imperfection of the senses; and what is still more remarkable, it follows the same course in the study of all phenomena; it interprets them by the same language, as if to attest the unity and simplicity of the plan of the universe, and to make still more evident that unchangeable order which presides over all natural causes.

The movement by mathematicians to build intricate representations of exotic mathematical destinations was paralleled by a wider 19th century educational movement to promote the use of much simpler models in the universities and public schools. Some of the currents in 19th century mathematics education reform that led to this development have been documented by Kidwell [**13**]. In the United States, for example, sets of geometric solids were sold to the newly established common schools in order to codify the impression of a common curriculum. The market for these models was maintained well into the early parts of the 20th century with fervent calls for, successively, "Object-Oriented Instruction", "Technical Training", "Art Education", and "Exact Thinking". The business eventually diversified into much more lucrative catalogues of "Mathematical Apparatus", which included everything from finely-crafted orreries and tellurians to the latest in elegant "Pointing Rods".

There was money to be made in outfitting mathematics departments with the instant educational cachet of a fine set of models, but there was also a genuine belief in their inherent instructional value, even among those who were doing the selling. This belief—that mathematical models could help to develop essential intuition about difficult analytic constructions—originated with two very influential mathematicians: Gaspard Monge in France and Felix Klein in Germany. Together, they set the standard for the way that mathematics was taught in Europe and America throughout the 19th century. Monge is known as the father of differential geometry, and his efforts in the early 1800s to classify surfaces by the motions of lines, along with his "descriptive geometry" for representing three-dimensional surfaces in two-dimensions, led naturally to the construction of elaborate models made of tightly stretched strings. One of his students, Théodore Olivier, built some of the most beautiful mathematical models ever made. He also made some money in the process: the models were expensive. Olivier sold them to the emerging technical schools in the United States, which were attempting to emulate the example of Monge and the École Polytechnic. Klein came along later in the century, promoting the use and construction of mathematical models in graduate education, and the first research universities in the United States did their best to follow the European lead. Klein and his colleague Alexander Brill established a Laboratory for the Construction of Mathematical Models in Munich, and the labors of their graduate students were reproduced and sold world-wide by Brill's brother Ludwig.

It was thus through a combination of free-spirited mathematical exploration, educational idealism, and conspicuous commercialism that the mathematical descendants of the *wunderkammern* came into being. Schools proudly displayed their newly acquired models, to show that they were up-to-date with the latest mathematical discoveries *and* the most progressive educational trends. The models were often housed in elaborately

Figure 3. Klein's Munich *Wunderkammern*.

crafted cabinets, made of fine wood. The models themselves sat on pedestals within the cabinets, sometimes on lush carpets of velvet. When they were taken from their case and into the classroom, they were presented, by all accounts, with great ceremony. Teachers and students making note of the Cartesian principles that the models were meant to demonstrate couldn't help but marvel at them—the treasures of the New World.

By the turn of the 20th century, mathematical *wunderkammern* had proliferated across Europe and America. Their spell, however, eventually lost its hold on the mathematical community. Economic realities in the early part of the 20th century made the acquisition of such "treasures" an increasingly difficult proposition, and the market for finely-crafted models, as well as the finely-crafted theories of education that went with them, fell off. Entrepreneurs jumped in, and for a while cheaper knock-offs supplied the diminishing demand. The increasingly clumsy constructions, however, could no longer capture the collective imagination. Mathematicians had gotten down to the hard work of sorting and classifying the accumulated discoveries of their great Age of Exploration, and traditional Cartesian work ethics were once again the mathematical vogue: skepticism, circumspection, and careful linear argument. Educators also gathered themselves together, returning to an emphasis on "fundamental skills". A purposefulness settled on the century, and dust began to settle on the *wunderkammern*.

4. WHITHER WONDER? The demise of the mathematical *wunderkammern* was fated, inevitably, by larger historical forces that had been in motion for centuries. The Enlightenment, with its emphasis on positivist inquiry and empirical science, had come rushing through the 19th century and into the 20th. Its great legacy, the Scientific Method, sought to draw careful boundaries around what had once been wonderful, charting intellectual continents of human dimension on a grid of comprehensible

Figure 4. Johns Hopkins University, ca. 1895.

design. The ability to map Nature became as much a source of wonder as Nature itself. Commerce became Industry, and the success of the Scientific Method encouraged the belief that everything inconvenient and unruly in Nature could eventually be outreasoned.

There were differences of opinion about this, of course. Perhaps nothing symbolized the opposing world-view so much as the Romantic movement in 19th century poetry, and perhaps no one symbolized that movement so much as Samuel Coleridge. During a period absent of muse, he wrote in his notebooks [**12**, p. 301]:

> In my long illness I had compelled into hours of Delight many a sleepless, painful hour of Darkness by chasing down metaphysical Game — and since then I have continued the Hunt, till I found myself unaware at the Root of Pure Mathematics — and up that tall smooth Tree, whose few poor branches are all at its very summit, am I climbing by pure adhesive strength of arms and thighs — still slipping down, still renewing my ascent. — You would not know me! — all sounds of similitude keep at such a distance from each other in my mind, that I have *forgotten* how to make a rhyme — I look at the Mountains (that visible God Almighty that looks in at all my windows) I look at the Mountains only for the Curves of their outlines; the Stars, as I behold them, form themselves into Triangles — and my hands are scarred with scratches from a Cat, whose back I was rubbing in the Dark in order to see whether the sparks were refrangible by a Prism. The Poet is dead in me — my imagination (or rather the Somewhat that had been imaginative) lies, like a Cold Snuff on the circular Rim of a Brass Candle-stick, without even a stink of Tallow to remind you that it was once cloathed and mitred with Flame. That is past by! — I was once a Volume of Gold Leaf, rising & riding on every breath of Fancy — but I have beaten myself back into weight and density, & now I sink in quicksilver, yea, remain squat & square on the earth amid the hurricane, that makes Oaks and Straws join in one Dance, fifty yards high in the Element.

There is, after all, something in the triumph of Science that is fundamentally opposed to the essential requirements of wonder. Wonder requires a diminished sense of oneself and one's capabilities. The historian Adalgisa Lugli has noted [**15**, p. 123] that:

> Wonder is a meta-historical category and extends up to the end of the eighteenth century. It is defined primarily in its didactic sense, as a form of learning—an intermediate, highly

particular state akin to a sort of suspension of the mind between ignorance and enlightenment that marks the end of unknowing and the beginning of knowing.

The great Age of mathematical *wunderkammern* was only a back-eddy in the larger currents of 19th century Science that were carrying the rest of the world over and around the "problems" in its path. It is not surprising that mathematicians, true to Science and forever beholden to manifestations of Cartesian progression, could not abide by their disorderly collections of curiosities for very long. They sought to understand the wonders, to put them in orderly mathematical contexts. After a century of dissection and classification, the elaborately constructed models now collecting dust in so many neglected cabinets look almost quaint, like the mementos of a faded romance.

Has wonder, then, finally disappeared beneath the flood of Science? The source of the flood, the rush toward Enlightenment and its ideals, has been diverted many times during the past century. The idea of Progress as Destiny has been battered about in political, economic, and social turbulence. In uncertain currents, the belief that Science would always find the true heading—implicit in Newton's mechanics, but not in Heisenberg's—has been called into question. Finally, and irrevocably, the twisted roots of Science were revealed in the brilliant light of the Atomic Bomb. As a result, uncertainty and disillusionment have shaped the past century as much as the ideals of the Enlightenment.

Wallace Stevens, the great philosophical poet of this past century, laments the legacy of living in the grip of a Method that is forever doomed by its inadequacies [**16**, p. 128]:

Oh! Blessed rage for order, pale Ramon,
The maker's rage to order words of the sea,
Words of the fragrant portals, dimly-starred,
And of ourselves and of our origins,
In ghostlier demarcations, keener sounds.

There is, however, something essentially *new* in this kind of uncertainty. Whereas in past centuries there may have been more general unknowing, this was accepted; it was not in conflict with previous experience. Before the Enlightenment, there was not the expectation that everything could be understood and controlled. Now, in the post-Enlightenment, the loss of certainty feels like the result of some sort of guilty excess, and there is an air of contrition surrounding every attempt to re-establish the Cartesian course. Mathematics has not been exempt: Attempts to control perceived excesses have consistently found their way into the debates on mathematics education. If the past century has revealed a fundamentally chaotic dynamic at the interface between Nature and the human mind, then (the arguments go, consciously or not) it must be banished from mathematics, in much the same way that the "misconceptions" of Galileo and Fourier were banished. The efforts are as futile now as they were then.

Like Lagrange, Abel, and Cauchy before them, there are mathematicians at the turn of this new millennium who recognize the fundamental sea change that has taken place, and are willing to accept it with wonder. While mathematics and science have undoubtedly helped to create the late 20th century aesthetic of containment and control, mathematicians and scientists also have a long history of fighting such strictures, especially when they begin to look like hegemony. Einstein didn't like the uncertainties of quantum mechanics, but he could still say that [**4**, p. 11]:

The most beautiful experience we can have is the mysterious. It is the fundamental emotion that stands at the cradle of true art and true science. Whoever does not know it can no longer wonder, no longer marvel, is as good as dead, and his eyes are dimmed.

Richard Feynman noted [**5**, p. 248]:

> It is our responsibility as scientists, knowing the great progress which comes from a satisfactory philosophy of ignorance, the great progress which is the fruit of freedom of thought, to proclaim the value of this freedom; to teach how doubt is not to be feared but welcomed and discussed; and to demand this freedom as our duty to all coming generations.

As long as there is "unknowing", there will be a recognition of the need for wonder.

In modern mathematics and mathematics education, the pervasive presence of "technological" curiosities is once again calling into question the Cartesian ideal of rigorously linear exposition. Computer investigations are inherently non-linear experiences, tossing the mathematical explorer about in a seemingly limitless sea of information. Even the most timid students of mathematics, however, can ably surf the Web. The numerical and graphical processing powers of computers have taken mathematicians to previously unimagined worlds, and allowed them to return with fantastic new treasures. There will continue to be, as there has always been, time to apply the lessons of history, and to sort, connect, and classify these marvelous new objects. It would seem a shame, however, with such fine galleons harbored on the desktops in almost every modern mathematics department, not to go along on the voyages, open to the wonder of what may lie ahead.

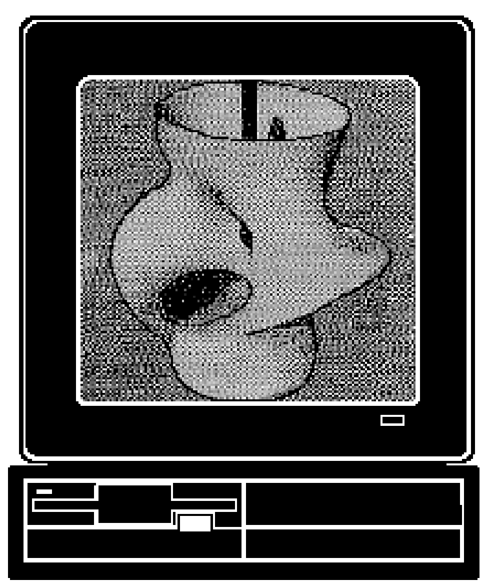

Figure 5. The Modern *Wunderkammern*.

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